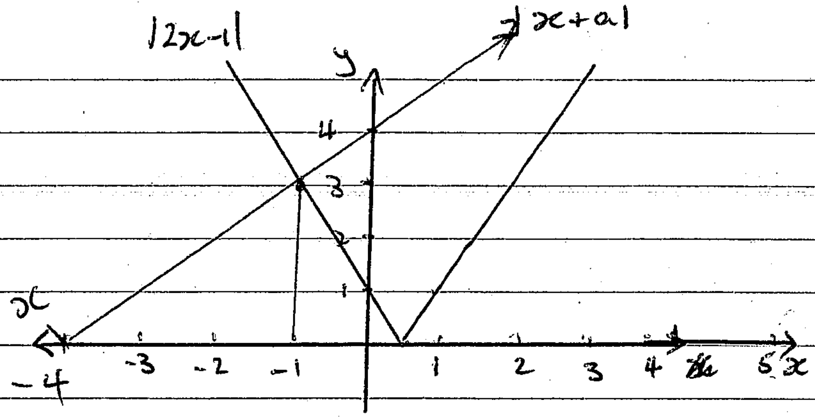
1a.



When $x = -1$,

$$|2x-1| = |x+a|$$
$$|-3| = |-1+a|$$
$$3 = |-1+a|$$
$$-1+a = 3 \quad \text{or} \quad -1+a = -3$$
$$a = 4 \quad \text{or} \quad a = -2$$

Since we're told $a > 0$, reject the second solution.

$\therefore$ $\underline{a = 4.}$

Check the other end of the interval at $x = 5$:

$$|2x-1| = |x+a|$$
$$|2\times 5-1| = |5+a|$$
$$9 = 5+4 \quad ✓$$

b. Choose a value for x in the interval and try the inequality.

E.g. $x = 0$

$$|2\times 0-1| \; ? \; |0+4|$$
$$|-1| \; ? \; |4|$$
$$1 < 4$$

We include the end points of the interval, so the ? ~~sign~~ must be $\underline{\underline{\leq}}$

c. 1st: $|x^2 - x - 21| = 39$

$$x^2 - x - 21 = 39 \quad \text{or} \quad x^2 - x - 21 = -39$$
$$x^2 - x - 60 = 0 \qquad x^2 - x + 18 = 0$$
$$x = \cancel{-6.59}\ -7.26 \quad \text{or} \quad x = \cancel{7.59}\ 8.26 \qquad \text{No sol}^n.$$

Next check either side of the inequality

eg $x = -8$, $x = 0$, $x = 9$

is $|(-8)^2 - (-8) - 21| < 39$?

$|64 + 8 - 21| = 51 \not< 39$ ✗

is $|0^2 - 0 - 21| < 39$?

$21 < 39$ ✓

is $|(9)^2 - 9 - 21| < 39$?

$|81 - 9 - 21| = 51 \not< 39$ ✗

Solution: $\underline{-7.26 < x < 8.26}$

Check by plotting $|x^2 - x - 21|$ on the classpad together with $y = 39$.

2. First solve the equation

$$|2x + 3| = |3x + 7|$$

$2x + 3 = 3x + 7$ or $2x + 3 = -(3x + 7)$

$x = -4$ $\qquad$ $x = \cancel{\tfrac{4}{8}}\ -2$

Next choose x to test

e.g. $x = 0$ (Not between -4 and -2)

is $|2(0) + 3| \leq |3(0) + 7|$?

Yes $(3 < 7)$

So the solution lies ~~between~~ outside the solutions to the equation, i.e.

$\{x \in \mathbb{R} : x < -4\}$ $\{x \in \mathbb{R}$ $x > -2\}$

3. First solve the equation

$$|x + 2| = \tfrac{1}{3}x + 1$$

$x + 2 = \tfrac{1}{3}x + 1$ or $x + 2 = -(\tfrac{1}{3}x + 1)$

$\tfrac{2}{3}x = -1$ $\qquad$ $x + 2 = -\tfrac{1}{3}x - 1$

$x = -\tfrac{3}{2}$ $\qquad$ $\tfrac{4}{3}x = -3$

$= -1.5$ $\qquad$ $x = -\tfrac{9}{4}$

$= -2.25$

Try x in between these values : $x = -2$

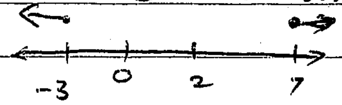
is $|-2 + 2| < \tfrac{1}{3}(-2) + 1$?

$0 < -\tfrac{2}{3} + 1$

$0 < \tfrac{1}{3}$ Yes

$\therefore \{x \in \mathbb{R} : -2.25 < x < -1.5\}$

4. Interpret the equation as "distance from a is 5 less than distance from 2".
This means a must be 5 units from 2: either 7 or -3.



If $a=7$ the solution set is $x \geq 7$; if $a=-3$ the solution set is $x \leq -3$

$\therefore \quad a=7, \quad b=7$

5.a $\angle B = 180 - 72 - 35$
$= 73^\circ$

$$\frac{AC}{\sin 73^\circ} = \frac{76}{\sin 72^\circ}$$

$$AC = \frac{76 \sin 73^\circ}{\sin 72^\circ}$$

$= 76.4$ cm

b $\quad \text{Area} = \frac{1}{2} a b \sin C$
$= \frac{1}{2} \times 10.7 \times 7.4 \sin 82^\circ$
$= 39.2 \text{ cm}^2$

6. $P \rightarrow B$ $\quad d = \frac{70}{360} \times 6350 \times 2\pi$

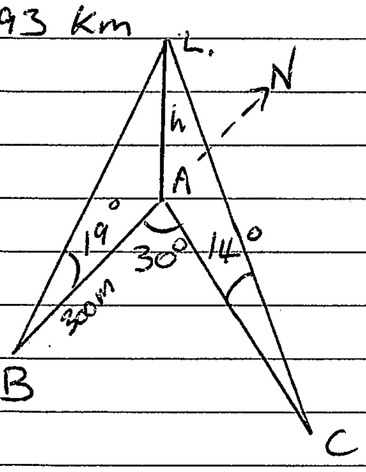
$= 7758$ km

$B \rightarrow A$ $\quad d = \frac{92}{360} \times 2\pi \times 6350 \cos 38^\circ$

$= 8035$ km

total distance $= 15793$ km

7a



b. $\tan 19^\circ = \frac{h}{300}$
$h = 300 \tan 19^\circ$
$= 103.3$ m

c. $\tan 14^\circ = \frac{h}{AC}$

$AC = \frac{h}{\tan 14^\circ}$
$= 414.3$ m

d. $BC^2 = 300^2 + 414.3^2 - 2 \times 300 \times 414.3 \cos 30^\circ$
$BC = 215.3$ m

8a. $AC = \sqrt{7^2 + 5^2 - 2\times7\times5\times\cos 1.4}$
$= 7.88\ m$

b. $\frac{AC}{\sin(\pi - 2.1 - \frac{33\pi}{180})} = \frac{8.3}{\sin 2.1}$

$\frac{AC}{0.4175} = 9.615$

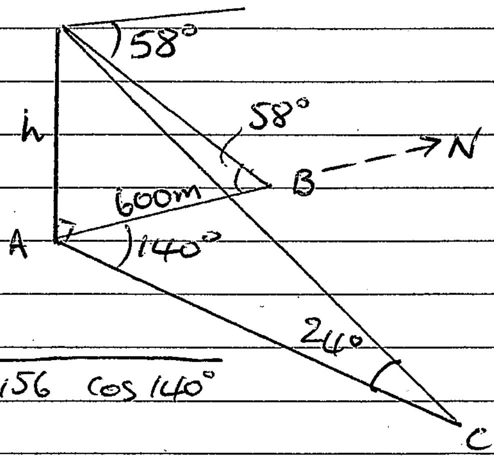
$AC = 0.4175 \times 9.615$
$= 4.01\ m$

c. Area $= \frac{1}{2} ab \sin C$
$= \frac{1}{2} \times \cancel{4.7}\,1.47 \times 7.4 \times \sin(180 - 82.9)$
$= \cancel{43.3}\ 4.33\ cm^2$

9. Travelling through $85°$ at $45°N$ latitude

$d = \frac{85}{360} \times 2\pi \times 6350 \cos 45°$

$= 6661\ Km$



10. a.
b. $h = 600 \tan 58°$
$= \underline{960\ m}$
c. $AC = \frac{960}{\tan 24°}$

$= 2156\ m$
$BC = \sqrt{600^2 + 2156^2 - 2\times600\times2156\ \cos 140°}$
$= \underline{\underline{2644\ m}}$

d. Angle is acute (since $\triangle ABC$ has an obtuse angle at A)
so sine rule is unambiguous.
$\frac{\sin B}{AC} = \frac{\sin A}{BC}$

$\sin B = \frac{2156 \sin 140}{2644}$

$B = 32°$

$\therefore$ Bearing of C from B is $180 - 32° = \underline{148°}$

11a Arc length $= \frac{360 - 66}{360} \times 2\pi \times 37 = 189.9$

$P = 189.9 + 2\times37 = 264\ mm$

11 b. $l = \frac{\theta}{360} \times 2\pi \times \cancel{\text{radius}}\, r$

$$5.54 = \frac{\theta}{360} \times 2\pi \times 4.66$$

$$\theta = \frac{360 \times 5.54}{2\pi \times 4.66}$$

$$= 68^\circ$$

12. a. $(6\sqrt{2} - 4\sqrt{3})(2\sqrt{3} + 3\sqrt{2}) = 12\sqrt{6} + 36 - 24 - 12\sqrt{6}$

$$= 12$$

b. $\sqrt{48} \times \sqrt{15} = \sqrt{(2^4 \times 3) \times (3 \times 5)}$

$$= 2^2 \times 3\sqrt{5}$$

$$= 12\,\cancel{144}\sqrt{5}$$

c. $\frac{3}{4\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{20}$

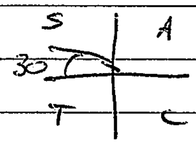
d. $\frac{\sqrt{7} - 1}{\sqrt{7} + 1} \; \frac{(\sqrt{7} - 1)}{(\sqrt{7} - 1)} = \frac{7 - 2\sqrt{7} + 1}{7 - 1}$

$$= \frac{8 - 2\sqrt{7}}{6}$$

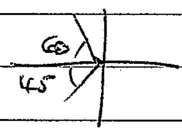
$$= \frac{4 - \sqrt{7}}{3}$$

13. a. $\tan 150^\circ = -\tan 30^\circ$

$$= -\frac{\sqrt{3}}{3}$$



b. $\frac{\cos 120^\circ}{\sin 225^\circ} = \frac{-\cos 60^\circ}{-\sin 45^\circ}$

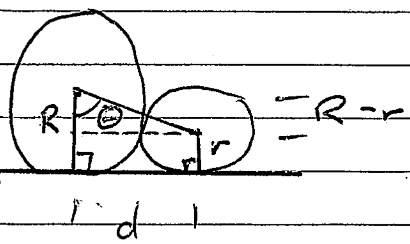


$$= \frac{1/2}{1/\sqrt{2}}$$

$$= \frac{1}{2} \times \frac{\sqrt{2}}{1}$$

$$= \frac{\sqrt{2}}{2}$$

14. $\cos\theta = \frac{R - r}{R + r}$



$$d = \sqrt{(R+r)^2 - (R-r)^2}$$

$$= \sqrt{R^2 + 2Rr + r^2 - (R^2 - 2Rr + r^2)}$$

$$= \sqrt{4Rr}$$

$$= 2\sqrt{Rr}$$

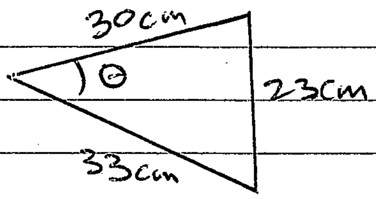
$A_{\text{TRAPEZIUM}} = \frac{1}{2}(R + r) \times d = (R + r)\sqrt{Rr}$

$$A_{sector\,A} = \frac{1}{2} R^2 \theta$$
$$= \frac{1}{2} R^2 \cos^{-1}\left(\frac{R-r}{R+r}\right)$$

$$A_{sector\,B} = \frac{1}{2} r^2 (\pi - \theta)$$
$$= \frac{1}{2} r^2 \left(\pi - \cos^{-1}\frac{R-r}{R+r}\right)$$

$$A_{SHADED} = A_{TRAPEZIUM} - A_{sector\,A} - A_{sector\,B}$$
$$= (R+r)\sqrt{Rr} - \frac{1}{2}R^2 \cos^{-1}\frac{R-r}{R+r} - \frac{1}{2}r^2\left(\pi - \cos^{-1}\frac{R-r}{R+r}\right)$$
$$= (R+r)\sqrt{Rr} - \frac{1}{2}(R^2 - r^2)\cos^{-1}\frac{R-r}{R+r} - \frac{\pi}{2}r^2$$

15. Smallest angle is opposite the shortest side.



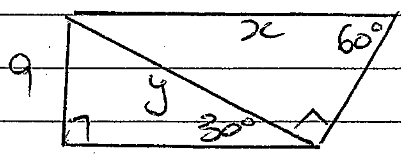
$$23^2 = 30^2 + 33^2 - 2 \times 30 \times 33 \cos\theta$$
$$\cos\theta = \frac{30^2 + 33^2 - 23^2}{2 \times 30 \times 33}$$
$$\theta = 49.49^\circ \quad (\text{or } 0.74^R)$$

16. a. i. $\cos 135^\circ = -\cos 45^\circ$
$$= -\frac{\sqrt{2}}{2} \quad \text{or} -\frac{1}{\sqrt{2}}$$

ii $\tan 210^\circ = \tan 30^\circ$
$$= \frac{1}{\sqrt{3}} \quad \text{or} \quad \frac{\sqrt{3}}{3}$$

b. $\sin^2 45^\circ + \cos^2 120^\circ = \left(\frac{1}{\sqrt{2}}\right)^2 + \left(-\frac{1}{2}\right)^2$
$$= \frac{1}{2} + \frac{1}{4}$$
$$= \frac{3}{4}$$

c. $\sin 30^\circ = \frac{9}{y}$

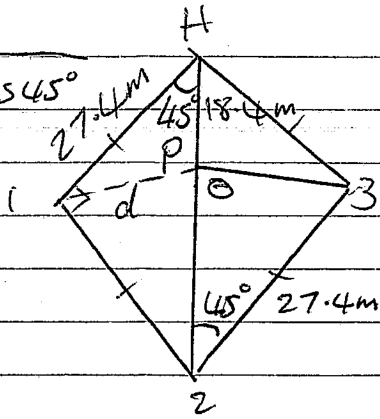


$$\frac{1}{2} = \frac{9}{y}$$
$$y = 18$$
$$\sin 60^\circ = \frac{y}{x}$$
$$x = \frac{y}{\sin 60^\circ}$$
$$= \frac{18}{\sqrt{3}/2}$$

$$x = \frac{36}{\sqrt{3}}$$
$$= \frac{36\sqrt{3}}{\sqrt{3}\,\sqrt{3}}$$
$$= 12\sqrt{3}$$

17.a $d = \sqrt{27.4^2 + 18.4^2 - 2 \times 27.4 \times 18.4 \cos 45^\circ}$

$= \underline{19.4m}$

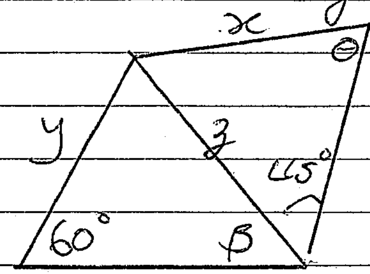


b. $\frac{\sin\theta}{27.4} = \frac{\sin 45}{19.4}$

$\theta = \sin^{-1} \frac{27.4 \sin 45}{19.4}$

$= \underline{87^\circ}$

NB: Unambiguous because we know θ is acute since PH is less than half the diagonal length (38.75m).



18. $\frac{z}{\sin\theta} = \frac{x}{\sin 45^\circ}$

$z = \frac{x \sin\theta}{1/\sqrt{2}}$

$= \sqrt{2}\, x \sin\theta$

$\frac{y}{\sin\beta} = \frac{z}{\sin 60^\circ}$

$y = \frac{z \sin\beta}{\sqrt{3}/2}$

$= \sqrt{2}\, x \sin\theta \sin\beta \times \frac{2}{\sqrt{3}}$

$= \frac{2\sqrt{2}\, x \sin\theta \sin\beta}{\sqrt{3}}$

19.a) $XM = \sqrt{OX^2 + (\tfrac{1}{2}AB)^2}$

$= \sqrt{60^2 + 25^2}$

$= 65\text{cm}$

$XB = \sqrt{XM^2 + MB^2}$

$= \sqrt{65^2 + 25^2}$

$= 69.64 \text{ cm}$

b) $\tan \angle XBA = \frac{MB}{XM}$

$\angle XBA = \tan^{-1} \frac{MB}{XM}$

$= \tan^{-1} \frac{25}{65}$

$= 21^\circ$

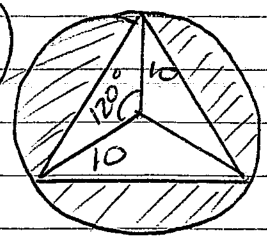
c) $\tan \angle XMO = \frac{XO}{OM}$

$= \frac{60}{25}$

$\angle XMO = \tan^{-1} \frac{12}{5}$

$= 67^\circ$

20. Area of triangle $= 3\left(\frac{1}{2} \times 10 \times 10 \sin 120^\circ\right)$



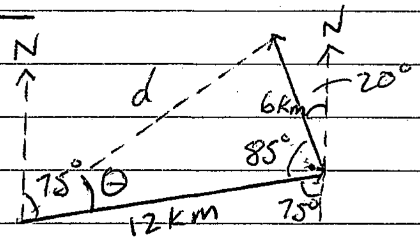
$= 129.90\ cm^2$

Area of circle $= \pi \times 10^2$

$= 314.16\ cm^2$

Shaded area $= 314.16 - 129.90$

$= 184.26\ cm^2$



21. a.

b. $d = \sqrt{12^2 + 6^2 - 2 \times 12 \times 6 \times \cos 85^\circ}$

$= 12.94$ km

c. $\frac{\sin \theta}{6} = \frac{\sin 85^\circ}{12.94}$

$\theta = \sin^{-1}\left(\frac{6 \sin 85^\circ}{12.94}\right)$

$= 27^\circ$

bearing $= (75 - \theta) + 180$

$= 227^\circ$

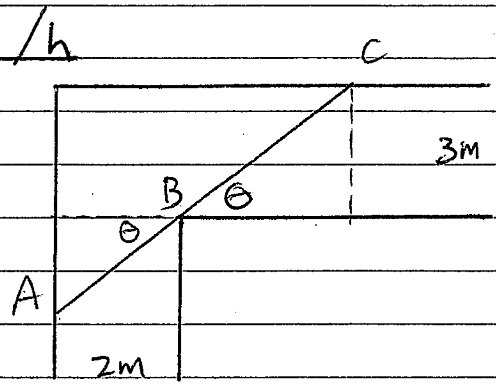
22. Circumference $= 2\pi R \cos 32^\circ$

$= 2\pi \times 6350 \times \cos 32^\circ$

$= 33836$ km

Speed $= \frac{33836}{24}$

$= 1410$ km/h



23. $\sin\theta = \frac{3}{BC}$

$BC = \frac{3}{\sin\theta}$

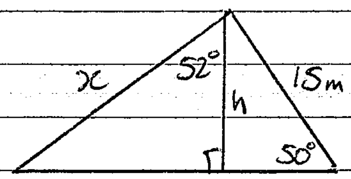
$\cos\theta = \frac{2}{AB}$

$AB = \frac{2}{\cos\theta}$

$AC = AB + BC$

$= \frac{2}{\cos\theta} + \frac{3}{\sin\theta}$ $\square$

24 a.



$\sin 50° = \frac{h}{15}$

$h = 15 \sin 50°$

$\cos 52° = \frac{h}{x}$

$x = \frac{h}{\cos 52°}$

$= \frac{15 \sin 50°}{\cos 52°}$

$= 18.66\text{m}$

b. $\tan 47° = \frac{a}{250\text{m}}$

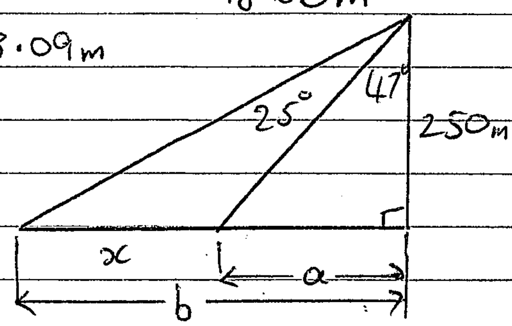
$a = 250 \tan 47°$

$= \cancel{12\,868\text{m}}\ 268.09\text{m}$

$\tan(25°+47°) = \frac{b}{250}$

$b = 250 \tan 72°$

$= 769.42\text{m}$

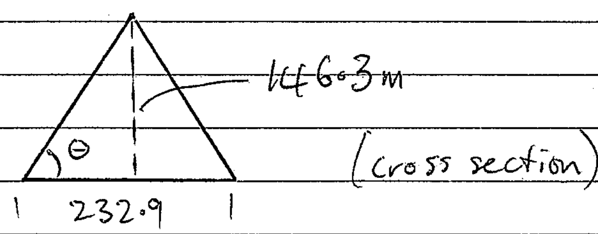


$x = b - a$

$= 501\text{ m}$

25. $\tan\theta = \frac{146.3}{0.5 \times 232.9}$

$\theta = 51°$



26. $A \to B$: $\frac{140}{360} \times 2\pi \times 6370 \cos 30°$

$= 13480\text{ km}$

$B \to C = \frac{50}{360} \times 2\pi \times 6370$

$= 5559\text{ km}$

$A \to C = 19038\text{ km}$

Time $= \frac{19038}{700} = 27.21$ hours.

At height of 10km:

$AB = \frac{140}{360} \times 2\pi \times 6380 \cos 30°$

$= 13501\text{ km}$

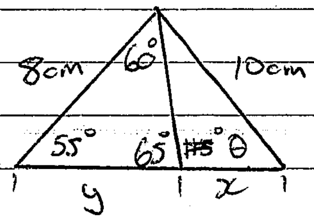
$BC = \frac{50}{360} \times 2\pi \times 6380$

$= 5568\text{ Km}$

$AC = 19068\text{ Km}$

Time $= \frac{19068}{700} = 27.24$ hours. (~2 min longer)

27a. $\dfrac{y}{\sin 60^\circ} = \dfrac{8}{\sin 65^\circ}$



$y = \dfrac{8 \sin 60^\circ}{\sin 65^\circ}$

$= 7.64 \text{ cm}$

$180 - 60 - 65 = 55^\circ$

$10^2 = 8^2 + (x+y)^2 - 2 \times 8 \times (x+y) \times \cos 55^\circ$

$x + y = 12.14$ (discarding negative root)

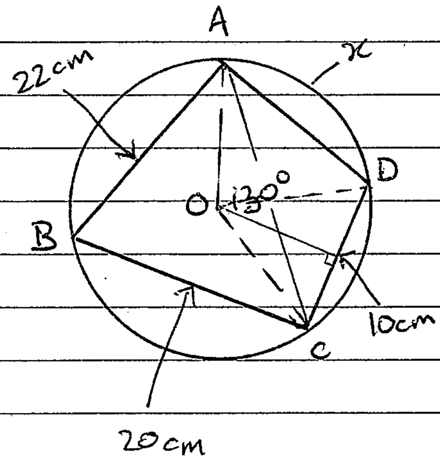
$x = 4.50 \text{cm}$

~~180 − 65 = 115°~~

$\dfrac{\sin 55}{10} = \dfrac{\sin\theta}{8}$

$\sin\theta = \dfrac{8 \sin 55}{10}$

$\theta = 41^\circ$



b. $\angle ABC = \dfrac{130}{2}$

$= 75^\circ$

$AC^2 = 22^2 + 20^2 - 2 \times 22 \times 20 \times \cos 75^\circ$

$AC = 25.6 \text{cm}$

$\sin \frac{130}{2} = \dfrac{AC/2}{r}$

$r = \dfrac{AC}{2 \sin 75^\circ}$

$= 13.3 \text{cm}$

$\sin \dfrac{\angle COD}{2} = \dfrac{10/2}{r}$

$\angle COD = 2 \sin^{-1} \dfrac{5}{r}$

$= 44.30^\circ$

$\angle AOD = 130 - \angle COD$

$= 85.7^\circ$

$x = \dfrac{\angle AOD}{360^\circ} \times 2\pi r$

$= 19.8 \text{ cm}$

28. Cos rule: $a^2 = b^2 + c^2 - 2bc\cos A$

rearrange to $-a^2 + b^2 + c^2 = 2bc\cos A$

similarly $a^2 - b^2 + c^2 = 2ac\cos B$

$a^2 + b^2 - c^2 = 2ab\cos C$

add LHS & RHS:

$a^2 + b^2 + c^2 = 2bc\cos A + 2ac\cos B + 2ab\cos C$

$= 2(bc\cos A + ac\cos B + ab\cos C)$

$\square$

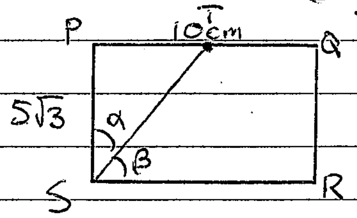
29. a. $\sqrt{32} = \sqrt{16\times 2}$
$= \sqrt{16} \times \sqrt{2}$
$= 4\sqrt{2}$

b. $\sqrt{10^2 - 5^2} = \sqrt{5^2\times 4 - 5^2}$
$= \sqrt{5^2 \times 3}$
$= 5\sqrt{3}$

c. $\sqrt{24} \times \sqrt{6} = \sqrt{4\times 6} \times \sqrt{6}$
$= 2\sqrt{6} \times \sqrt{6}$
$= 2 \times 6$
$= 12$

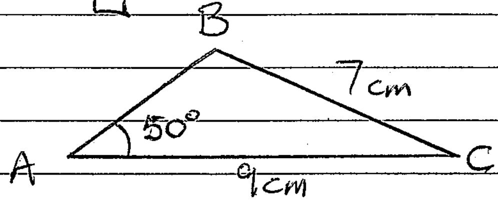
d. $\dfrac{\sqrt{6}}{\sqrt{10} - \sqrt{6}} = \dfrac{\sqrt{6}\,(\sqrt{10} + \sqrt{6})}{(\sqrt{10} - \sqrt{6})(\sqrt{10} + \sqrt{6})}$
$= \dfrac{\sqrt{2}\sqrt{3}\sqrt{2}\sqrt{5} + 6}{10 - 6}$
$= \dfrac{2\sqrt{15} + 6}{4}$
$= \dfrac{\sqrt{15} - 3}{2}$

30. $\tan\alpha = \dfrac{PT}{PS}$
$= \dfrac{5}{5\sqrt{3}}$
$= \dfrac{1}{\sqrt{3}}$



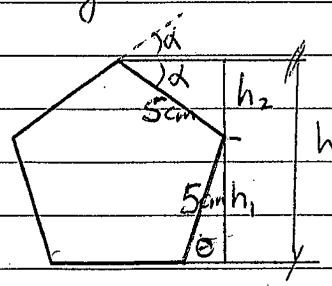
$\alpha = 30^\circ$
$\beta = 60^\circ$
$\angle TRS = 60^\circ$ (by symmetry)
$\therefore \angle STR = 60^\circ$
$\triangle TSR$ is equilateral. □

31. $\dfrac{\sin B}{9} = \dfrac{\sin 50}{7}$



$B = \sin^{-1} \dfrac{9\sin 50^\circ}{7}$
$= 80^\circ$
$C = 180 - 80 - 50$
$= 50^\circ$ ⇒ triangle is isosceles
$c = 7$ cm.

32. $\theta = \frac{360}{5} = 72^\circ$



$h_1 = 5\sin 72^\circ$
$= 4.76$ cm
$\alpha = 36^\circ$
$h_2 = 5\sin 36^\circ$
$= 2.94$ cm
$h = 4.76 + 2.94$
$= 7.69$ cm

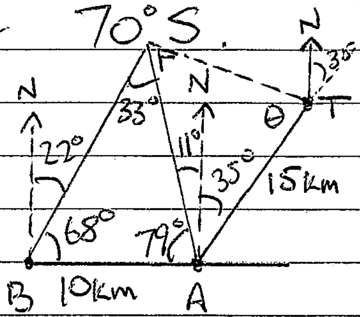
33. $d = \frac{\theta}{360} \times 2\pi \times 6350 \cos(lat.)$

$1200 = \frac{32}{360} \times 2\pi \times 6350 \cos(lat.)$

$\cos(lat) = \frac{1200 \times \frac{360}{32}}{2\pi \times 6350}$

$lat. = 70^\circ N$ or $70^\circ S$.

34. $180 - 68 - 79 = 33^\circ$



$\frac{AF}{\sin 68^\circ} = \frac{10}{\sin 33^\circ}$

$AF = \frac{10 \sin 68^\circ}{\sin 33^\circ}$

$= 17.02 \text{ km}$

$TF^2 = 15^2 + 17.02^2 - 2 \times 15 \times 17.02 \times \cos(11 + 35)$

$TF = 12.65 \text{ km}$

$\cos\theta = \frac{15^2 + 12.65^2 - 17.02^2}{2 \times 15 \times 12.65}$

$\theta = 75.5^\circ \qquad (75.47^\circ)$

bearing $= 35 + 180 + 75$

$= 290^\circ$

35. i $AOP = \pi - \theta$

ii $\text{Area} = \frac{1}{2} r^2 (\alpha - \sin\alpha)$

$= \frac{1}{2} AO^2 (\pi - \theta - \sin(\pi - \theta))$

36. a. $l = r\theta$

$= 163 \times 2.2$

$= 358.6 \text{m}$

$\approx 359 \text{m}$

b. $P = 2 \times 37 + \frac{360 - 66}{360} \times 2 \times 37 \times \pi$

$= 263.9 \text{ mm}$

$\approx 264 \text{ mm}$

c. $l = r\theta$

$4.66\theta = 5.54$

$\theta = 1.189^R$

$= 1.189 \times \frac{180}{\pi}$

$= 68^\circ$

d. $P = 2r + r\theta$

$10 = r(2 + 1.5)$

$r = \frac{10}{3.5}$

$= 2.86 \text{m}$

37a. $a = \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 42^2 \times 1.20$

$= 1058.4 \text{ cm}^2$

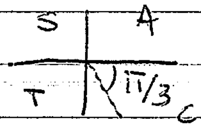
b. $a = \frac{1}{2} r^2 (\theta - \sin\theta)$

$= \frac{1}{2} \times 150^2 (0.8 - \sin 0.8)$

$= 930 \text{ mm}^2$

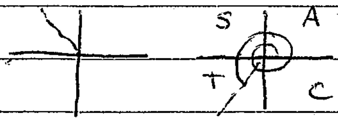
38. $\frac{65}{180}\pi = \frac{13\pi}{36}$

39\. a. $\tan \frac{5\pi}{3} = -\tan \frac{\pi}{3}$



$= -\sqrt{3}$

b. $\sin \frac{-3\pi}{4} - \cos \frac{13\pi}{4}$



$= \sin \frac{\pi}{4} - \left(-\cos \frac{\pi}{4}\right)$

$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$

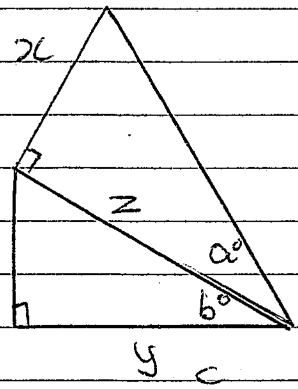
$= \sqrt{2}$

40 $\tan a = \frac{x}{z}$

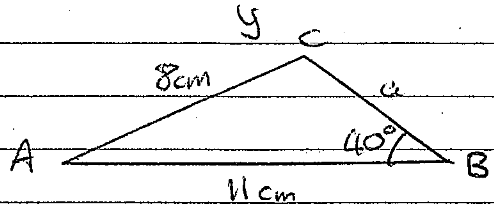
$z = \frac{x}{\tan a^\circ}$

$y = z \cos b^\circ$

$= \frac{x \cos b^\circ}{\tan a^\circ}$



41\. $\frac{\sin C}{11} = \frac{\sin 40^\circ}{8}$



$\sin C = \frac{11 \sin 40^\circ}{8}$

$\sin C = 0.8838$

$C = 62^\circ$ or $C = 180 - 62 = 118^\circ$

$A = 180 - 62 - 40$ | $A = 180 - 118 - 40$

$= 78^\circ$ | $A = 22^\circ$

$\frac{a}{\sin 78^\circ} = \frac{8}{\sin 40^\circ}$

$a = \frac{8 \sin 78^\circ}{\sin 40^\circ}$ | $a = \frac{8 \sin 22^\circ}{\sin 40^\circ}$

$= 12.2$ cm | $= 4.7$ cm

42 $p = 2r + r\theta$

$40 = 2 \times 10 + 10\theta$

$\theta = 2$

$A = \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 10^2 \times 2$

$= 100 \text{ cm}^2$

43a. $a = \frac{30}{360} \times \pi r^2$

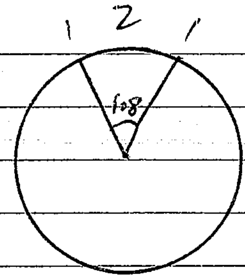
$= \frac{1}{12} \times \pi \times 5^2$

$= \frac{25\pi}{12} \text{ cm}^2$

$\approx 6.54 \text{ cm}^2$

b. $A = \frac{1}{2} \times 3.5^2 \left(\frac{\pi}{4} - \sin \frac{\pi}{4}\right)$ or $A = \frac{1}{2} r^2 (\theta - \sin\theta)$

$= 0.480 \text{ cm}^2$



43c. $r\theta = 2$

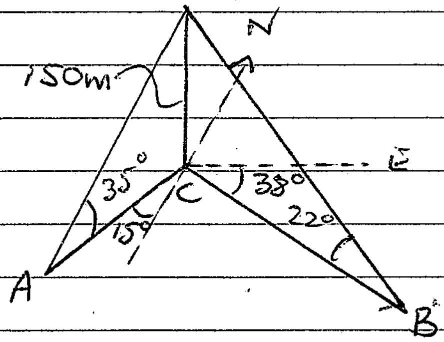
$r = \frac{2}{1.8}$

$= 1.11$

$a = \frac{1}{2} r^2 (2\pi - \theta)$

$= \frac{1}{2} \times 1.11 \, (2\pi - \theta)$

$= 2.49 \text{ cm}^2$



44. $\tan 35° = \frac{150}{AC}$

$AC = \frac{150}{\tan 35°}$

$= 214 \text{ m}$

$\tan 22° = \frac{150}{BC}$

$BC = \frac{150}{\tan 22°}$

$= 371 \text{ m}$

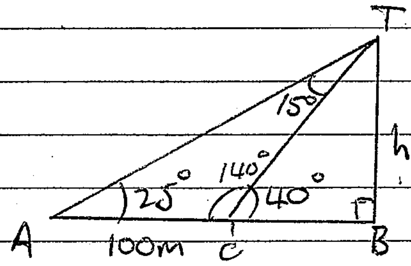
$\angle ACB = 90 - 38 + 15$

$= 67°$

$AB = \sqrt{AC^2 + BC^2 - 2AC \times BC \sin 67°}$

$= \sqrt{214^2 + 371^2 - 2 \times 214 \times 371 \times \sin 67°}$

$= 349 \text{ m}$



45. $\frac{CT}{\sin 25°} = \frac{100}{\sin 15°}$

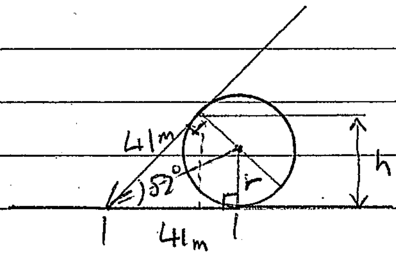
$CT = \frac{100 \sin 25°}{\sin 15°}$

$= 163.3 \text{ m}$

$\sin 40° = \frac{h}{163.3}$

$h = 163.3 \sin 40°$

$= 105 \text{m}$



46. $\tan 26° = \frac{r}{41}$

$r = 41 \tan 26°$

$= 20 \text{ m}$

$\sin 52° = \frac{h}{41\text{m}}$

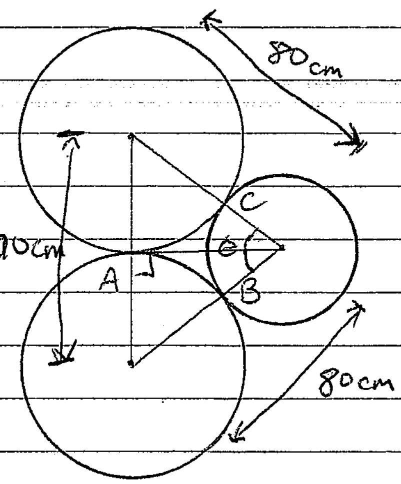
$h = 41 \sin 52°$

$= 32.3 \text{ m}$

47. $\sin\frac{\theta}{2} = \frac{50}{80}$

$\frac{\theta}{2} = \cancel{38.68°}\ 0.675$

$\theta = \cancel{77}\ 1.350$



Area of $\Delta = \frac{1}{2}ab\sin C$

$= \frac{1}{2}80^2 \times \sin 1.350$

$= 3122.5\ cm^2$

Area small sector: $\frac{1}{2}r^2\theta$

$= \frac{1}{2}\times 30^2 \times 1.35$

$= 607.6\ cm^2$

Area large sector $= \frac{1}{2}r^2\theta$

$= \frac{1}{2}\times 50^2 \times \left(\frac{\pi - 1.35}{2}\right)$

Area 2 large sectors $= 50^2 \times \frac{\pi - 1.35}{2}$

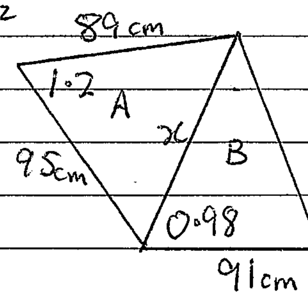
$= 2239.2$

Area enclosed $= 3122.5 - 607.6 - 2239.2$

$= 275.7\ cm^2$

48. $x = \sqrt{89^2 + 95^2 - 2\times 89 \times 95 \times \cos 1.2}$

$= 104\ cm$



area A $= \frac{1}{2}ab\sin C$

$= \frac{1}{2}\ 89\times 95 \times \sin 1.2$

$= 3940\ cm^2$

area B $= \frac{1}{2}\times 104 \times 91 \times \sin 0.98$

$= 3930\ cm^2$

Total area $= 7871\ cm^2$

49a $\overrightarrow{AB} = \begin{pmatrix} 9 \\ -2 \end{pmatrix}$

b. $|\overrightarrow{AB}| = \sqrt{9^2+2^2}$

$= \sqrt{85}$

Unit vector $= \frac{1}{\sqrt{85}}\begin{pmatrix} 9 \\ -2 \end{pmatrix}$

c. $\overrightarrow{AB} + \frac{3}{2}\overrightarrow{OB} = \begin{pmatrix} 9 \\ -2 \end{pmatrix} + \frac{3}{2}\begin{pmatrix} 6 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 9 \\ -2 \end{pmatrix} + \begin{pmatrix} 9 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 18 \\ 1 \end{pmatrix}$

$\left|\begin{pmatrix} 18 \\ 1 \end{pmatrix}\right| = \sqrt{18^2+1^2}$

$= \sqrt{325}$

$= 5\sqrt{13}$

d. $\begin{pmatrix} 6 \\ y \end{pmatrix} = k\begin{pmatrix} 9 \\ -2 \end{pmatrix}$

$k = \frac{2}{3}$ $\quad y = -\frac{4}{3}$

50 a. $\overrightarrow{OA} = \underline{a} + \underline{b}$

b. $\overrightarrow{AB} = \underline{a} - \underline{b}$

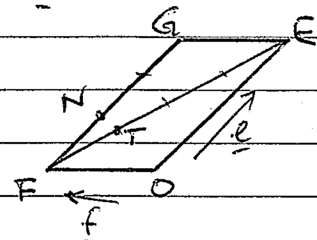
c. $\overrightarrow{BC} = \underline{a} + 4\underline{b}$

d. $\overrightarrow{CO} = -3\underline{a} - \underline{b}$

51. a. $\underline{r}_a = 11\underline{i} + 4\underline{j} + 0.5(2\underline{i} - 16\underline{j}) = 12\underline{i} - 4\underline{j}$

b. $\underline{r}_{A(t)} = (12\underline{i} - 4\underline{j}) + t(2\underline{i} - 16\underline{j})$ km

$\underline{r}_{B(t)} = (-22\underline{i} - 55\underline{j}) + t(10\underline{i} - 4\underline{j})$ km.



52. a. i. $\overrightarrow{FE} = -\overrightarrow{OF} + \overrightarrow{OE}$

$= \underline{e} - \underline{f}$

ii $\overrightarrow{ON} = \overrightarrow{OF} + \frac{1}{3}\overrightarrow{FG}$

$= \underline{f} + \frac{1}{3}\underline{e}$

iii $\overrightarrow{OT} = \overrightarrow{OF} + \frac{1}{4}\overrightarrow{FE}$

$= \underline{f} + \frac{1}{4}(\underline{e} - \underline{f})$

$= \frac{3}{4}\underline{f} + \frac{1}{4}\underline{e}$

b. Collinear if $\overrightarrow{ON} = k\,\overrightarrow{OT}$

$\underline{f} + \frac{1}{3}\underline{e} = k\left(\frac{3}{4}\underline{f} + \frac{1}{4}\underline{e}\right)$

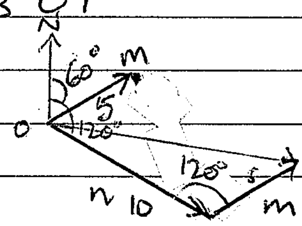
$\underline{f}$ - component: $1 = k \times \frac{3}{4}$

$k = \frac{4}{3}$

$\underline{e}$ - component $\frac{1}{3} = k \times \frac{1}{4}$

$k = \frac{4}{3}$

$\therefore \overrightarrow{ON} = \frac{4}{3}\overrightarrow{OT}$ □



53. $\underline{m} = 5\sin 60°\underline{i} + 5\cos 60°\underline{j}$

$= \frac{5\sqrt{3}}{2}\underline{i} + \frac{5}{2}\underline{j}$

$\underline{n} = 10\sin 120°\underline{i} + 10\cos 120°\underline{j}$

$= 10\sin 60\underline{i} + -10\cos 60\underline{j}$

$= \frac{10\sqrt{3}}{2}\underline{i} - 5\underline{j}$

$\underline{m} + \underline{n} = \frac{15\sqrt{3}}{2}\underline{i} - \frac{5}{2}\underline{j}$

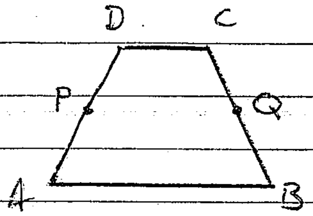
$|\underline{m} + \underline{n}| = \sqrt{\left(\frac{15\sqrt{3}}{2}\right)^2 + \left(\frac{5}{2}\right)^2}$

$= 5\sqrt{7}$

$\tan\theta = \dfrac{\frac{15\sqrt{3}}{2}}{-\frac{5}{2}} = -3\sqrt{3}$

$\theta = 101°$

$\underline{m} + \underline{n} = 5\sqrt{7}$ units on a bearing of $101°$.



54 $\vec{PQ} = \vec{PD} + \vec{DC} + \vec{CQ}$

$= \frac{1}{2}\vec{AD} + \vec{DC} - \frac{1}{2}\vec{BC}$ ①

$\vec{PQ} = \vec{PA} + \vec{AB} + \vec{BQ}$

$= -\frac{1}{2}\vec{AD} + \vec{AB} + \frac{1}{2}\vec{BC}$ ②

$2\vec{PQ} = \vec{AB} + \vec{DC}$ ①+②

$\vec{PQ} = \frac{1}{2}(\vec{AB} + \vec{DC})$

Since $\vec{AB} \parallel \vec{DC}$, $\vec{AB} + \vec{DC}$ is a scalar multiple of $\vec{AB}$ (or $\vec{DC}$)

and hence $\vec{PQ} \parallel AB \parallel DC$

and also $|\vec{AB} + \vec{DC}| = |\vec{AB}| + |\vec{DC}|$

hence $|\vec{PQ}| = \frac{1}{2}(|\vec{AB}| + |\vec{DC}|)$

55. Plot $A(-4,1)$, $B(2,6)$

$r_C = {}_C r_B + r_B = -7i - 3j$ Plot C at $(-7,-3)$

$r_D = {}_D r_A + r_A = -8i - j$ D $(-8,-1)$

${}_B r_E = r_B - r_E \Rightarrow r_E = r_B - {}_B r_E = -2i + 12j$ E $(-2,12)$

${}_F r_C = {}_A r_B = r_A - r_B = -6i - 5j$

$r_F = {}_F r_C + r_C = (-6i - 5j) + (-7i - 3j) = -13i - 8j$ F $(-13,-8)$

${}_G r_B = {}_D r_B + {}_B r_A = {}_D r_A = (-8i - j) - (-4i + j) = -4i - 2j$

$r_G = {}_G r_B + r_B = (-4i - 2j) + (2i + 6j) = (-2i + 4j)$

plot G $(-2,4)$

56. LHS: ${}_A r_C - {}_B r_C = (r_A - r_C) - (r_B - r_C)$

$= r_A - r_C - r_B + r_C$

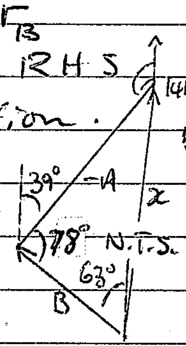
$= r_A - r_B$

$= {}_A r_B$

$=$ RHS □

57. Classpad construction.

$180 - 39 - 63 = 78°$



${}_B V_A = {}_{\cancel{B}} V_B - V_A$

$= V_B + -V_A$

$= 34.86$ m/s on a bearing of 001°

58 $x^2 = 32^2 + 22^2 - 2 \times 32 \times 22 \cos 78°$

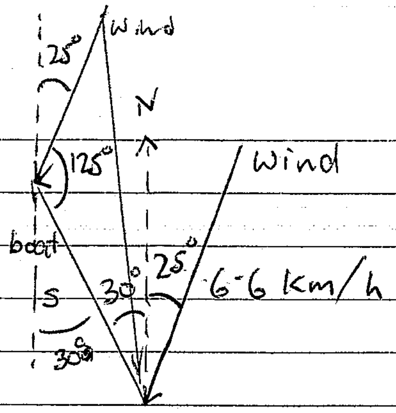
$x = 34.86$

$\frac{\sin\theta}{22} = \frac{\sin 63°}{32}$

$\theta = \sin^{-1}\left(\frac{22 \sin 63°}{32}\right)$ | $39 - 37.8 = 1.2°$

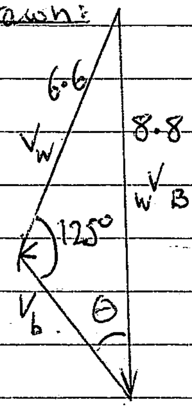
$= 37.8°$

First sketch



59 $|{}_W V_B| = 8.8$ km/h

Redrawn:



a. Let the speed of the boat be S.

$8.8^2 = 6.6^2 + S^2 - 2 \times 6.6 \times S \cos 125^2$

$S = -10.7$ or 3.16

discard negative root.

Speed of boat is 3.2 km/h

b. $\dfrac{\sin\theta}{6.6} = \dfrac{\sin 125}{8.8}$

$\theta = \sin^{-1} \dfrac{6.6 \sin 125}{8.8}$

$= 38°$

Wind direction $= 38 - 30$

$= 008°$.

60. a. $3^x = 5$

$\log 3^x = \log 5$ or simply $x = \log_3 5$

$x \log 3 = \log 5$ $= 1.46$

$x = \dfrac{\log 5}{\log 3}$

$= 1.46$

b. $\log_2 (2x+1) - \log_2 x = 2$

$\log_2 \dfrac{2x+1}{x} = 2$

$\dfrac{2x+1}{x} = 2^2$

$= 4$

$2x+1 = 4x$

$1 = 2x$

$x = \frac{1}{2}$

61. $2\log_2 x + \log_2 (x-1) = 1 + \log_2 (5x+4)$

$\log_2 x^2 + \log_2 (x-1) = \log_2 2 + \log_2 (5x+4)$

$\log_2 (x^2(x-1)) = \log_2 (2(5x+4))$

$x^2(x-1) = 2(5x+4)$

$x^3 - x^2 = 10x + 8$

$x^3 - x^2 - 10x - 8 = 0$ $\square$

62 a. $5^{x-2} = 29$

$\log_5 5^{x-2} = \log_5 29$

$x-2 = \log_5 29$

$x = \log_5 29 + 2$

$= 4.09$

b. $4^{3x} = 5^{x-1}$

$3x\log 4 = (x-1)\log 5$

$(3\log 4)x = (\log 5)x - \log 5$

$(3\log 4 - \log 5)x = -\log 5$

$x = \dfrac{\log 5}{\log 5 - 3\log 4}$

$= -0.631$

63 $\dfrac{3^{2x+1} - 3^2}{3^{2x-1} - 3^0} = \dfrac{3^1(3^{2x} - 3^1)}{3^{-1}(3^{2x} - 3^1)}$

$= \dfrac{3^1}{3^{-1}}$

$= 3^2$

$= 9$

64. a. $\log_3 27 = \log_3 3^3 = 3$

b. $\log_2 2^4 = 4$

Not in course

65 a. $32e^{0.02\times 12} = 40\,680\,000$

b. $32e^{0.02\times 2} - 32e^{0.02\times 1} = 659502$

c. $P_E = 60e^{-0.025t}$ (or $P_E = 60(0.975)^t$)

d. Solve $60e^{-0.025x} = 30$

$x = 27.7$

Pop^n will fall to 30 million in the 28th year, i.e. 2028.

e. Solve $60e^{-0.025x} = 32e^{0.02x}$

$x = 14.0$ (13.97)

Pop^{n}s will be equal at the end of 2013.

66. a. $\log y = 3\log x - \log x^2$

$= 3\log x - 2\log x$

$= \log x$

$y = x$

(b.) $2\log_5 y = \log_5 4x^2 + 2\log_5 x$

$= \log_5 (2x)^2 + 2\log_5 x$

$= 2\log_5 2x + 2\log_5 x$

$\log_5 y = \log_5 2x + \log_5 x$

$= \log_5 2x^2$

$y = 2x^2$

67. Assume $y = ka^x$

$\frac{162}{150} = 1.08$ $\frac{175}{162} = 1.08$ $a = 1.08$, $k = 150$

$\frac{189}{172} = 1.08$

a. 10^{th} day $\Rightarrow x = 9$ $y = 150\times 1.08^9 = 300$ people

67 b. $150 \times 1.08^{x} = 1000$

$x = 24.65$

Over 1000 diagnosed after 25 days (i.e. on the 26^{th} day).

68 a. $f \circ g(x) = f(x^3)$
$= \frac{3}{x^3}$

(b.) $g(h(t)) = g(\sin x)$
$= \sin^3 x$

c. $f \circ g \circ h\left(\frac{-\pi}{2}\right) = f \circ g\left(\sin -\frac{\pi}{2}\right)$
$= f \circ g(-1)$
$= f(-1^3)$
$= f(-1)$
$= \frac{3}{-1}$
$= -3$

d. $h \circ g \circ f(3) = h \circ g\left(\frac{3}{3}\right)$
$= h \circ g(1)$
$= h(1^3)$
$= h(1)$
$= \sin(1)$
$= 0.84$

e $(f \circ g)^{-1} =$

$y = \frac{3}{x^3}$
$x^3 = \frac{3}{y}$
$x = \sqrt[3]{3/y}$

$(f \circ g)^{-1}(x) = \sqrt[3]{\frac{3}{x}}$

69 $h \circ k(x) = h(x^2)$
$= \sqrt{x^2 - 1}$

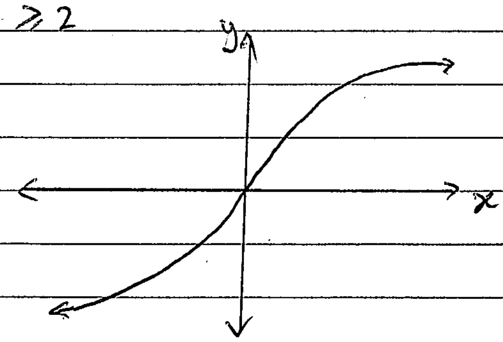
Natural domain : $x^2 - 1 \geq 0$
$x^2 \geq 1$

$x \geq 1$ or $x \leq -1$

However given that $k(x)$ is only defined for $x \geq 0$ we would limit domain of $h \circ k(x)$ to $x \geq 1$.

70 a. $y = 6 - (x-2)^2$, $x \geq 2$
$(x-2)^2 = 6 - y$
$x - 2 = \sqrt{6-y}$
$x = \sqrt{6-y} + 2$
$f^{-1}(x) = \sqrt{6-x} + 2$

b.



71a. Fails horizontal line test.

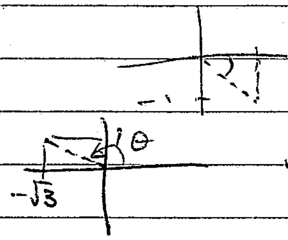
b. $x \geq 2$ $\quad$ [or $x \leq 2$]

c. $\{x \in \mathbb{R};\ x \geq 0\}$, $\{y \in \mathbb{R};\ y \geq 2\}$ [or $\{y \in \mathbb{R};\ y \leq 2\}$]

72 a. $A: (\cos\alpha, \sin\alpha)$ $\qquad$ $B: (3\cos\beta, 3\sin\beta)$

b. $C: \left(\sqrt{2}, -\frac{\pi}{4}\right)$

$D \ \left(2, \frac{5\pi}{6}\right)$



$\sqrt{\sqrt{3}^2 + 1^2} = 2$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\theta = -\frac{\pi}{6}$ ✗

or $\theta = \frac{5\pi}{6}$ ✓

73. $B: (5\cos(-2), 5\sin(-2))$

$A: (5, -2)$

$AB = \sqrt{(5\cos(-2) - 5)^2, (-2 - 5\sin(-2))^2}$

$= 18 \cdot 03.$

74.